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Summary Sheet

Abstract

We establish two models to analyze the traffic flow mixed with self-driving cars (**SDV**) and non-self-driving cars (**NSDV**): Modern Dynamic Model of Traffic Flow (**MDTF**) and Smart Traffic Flow Model Based on Cellular Automata (**STCA**)

Our first model is based on the conservation laws in hydrodynamics. We introduce a new variable: the proportion of SDVs and we analyze two typical situations: the diffusion of traffic jam and the effects of ramps. According to the simulation results by difference method, we conclude that SDVs can lighten the diffusion of traffic jam and smooth the density distribution of traffic flow.

Our second model is based on one dimensional CA. We firstly divide the information a vehicle may receive into two types: in-horizon information (**IHI**) and out-horizon information (**OHI**). Based on this, we abstract two laws (moving and changing lanes) to depict the mechanism of traffic flow mixed with SDVs, where we not only consider the cooperation between SDVs, but also consider the interaction between SDVs and NSDVs. Moreover, we define a synchronization effect between two SDVs and we proposed four indexes to evaluate the effects of SDVs on traffic flow: average velocity of vehicles (**AV**), rate of low-speed vehicles (**RLV**), frequency of slamming on breaks (**FSB**) and frequency of changing lanes (**FCL**). Finally, we fit the function between each index and the proportion of SDVs and we find that they will all drop evidently with increasing number of SDVs.

To apply our model to real data, we combine location data in Excel spreadsheet with speed data in ***Washington State Speed Report***. Then we compare the simulation results with real data and predict the effect of SDVs. Significantly, we offer some valuable suggestion to the governor of Washington State in the final letter!

Key words: MDTF, STCA, IHI, OHI, AV, RLV, FSB, FCL, SDV, NSDV

Contents

1. A Literature Review on Traffic Flow Models

Transportation system is of great significance to the development of human society and community economy in contemporary world and a highly efficient transportation system can accelerate the manufacturing development of a country remarkably. However, with the increasing number of vehicles in the whole world, traffic jam has become a worldwide problem, which promotes the development of traffic flow theory.

In general, there are two main research areas of traffic flow theory: the first one is dynamic traffic flow model based on the conservation laws of hydrodynamics in a macroscopic perspective and the second one is discrete model based on the theory of cellular automata in a microscopic perspective.

The first dynamics traffic flow model (LWR model) was proposed by Lighthill and Whitham [1] in 1955, who introduced continuity equation of traffic flow. This equation can be written as follows:

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho v}{\partial x} = g(x,t)\,, \tag{1.1}$$

where ρ is the density of traffic flow, v is the velocity of traffic flow and $g(x,t)$ is a source item. Though LWR model is easy to find numerical solution, it cannot simulate complicated traffic system and explain real traffic phenomena well.

Based on LWR model, Payne [2] established the momentum equation of traffic flow in 1971, which has the following form:

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} = -\frac{\gamma}{\rho T}\frac{\partial \rho}{\partial x} + \frac{v_e - v}{T}\,, \tag{1.2}$$

where T is the relaxation time of vehicle, γ is the expectation index and v_e is the balance velocity of traffic flow. The first item on the right side is expectation item, which depicts the reaction process of drivers when adjusting the velocity of vehicle according to the traffic flow. The second item on the right side is relaxation item, which reflects the adjustment process of drivers to make the vehicle reach balance velocity. Though Payne's model can describe the traffic flow more accurately compared with LWR model, Daganzo [3] proposed that there existed backward motion problems in this model.

Classical car-following model has the following form [4]:

$$\frac{dv_{n+1}(t+\Delta t)}{dt} = \lambda \Delta v, \tag{1.3}$$

where $\Delta v = v_n(t) - v_{n+1}(t)$, $v_n(t)$ is the velocity of the front car, $v_{n+1}(t)$ is velocity of the following car, Δt is the reaction time of drivers and λ is a reaction coefficient.

Bando, et al [5] proposed OVM model, where they assumed that the following car's state changed not only with the difference between the velocity of the front car and the velocity of the following car, but also with the difference between the location of the front car and the location of the following car. OVM model has the following form:

$$\frac{\mathrm{d}v_{n+1}(\mathrm{t})}{\mathrm{dt}} = \kappa[V(\Delta x) - v_{n+1}(\mathrm{t})] \tag{1.4}$$

where κ is a reaction coefficient and V is a function about x.

Based on OVM model, Helbing, et al [6] proposed GFM model. They assumed that when $\Delta v < 0$, $\lambda \Delta v$ needed to be considered as a factor which affected the change of velocity, which can be written as follows:

$$\frac{\mathrm{d}v_{n+1}(\mathrm{t})}{\mathrm{dt}} = \kappa[V(\Delta x) - v_{n+1}(\mathrm{t})] + \lambda \Delta v H(-\Delta v) \tag{1.5}$$

$$H(x) = \begin{cases} 1, & x \geq 0 \\ 0, & x < 0 \end{cases}. \tag{1.6}$$

Later, Rui Jiang [7] proposed that when $\Delta v > 0$, $\lambda \Delta v$ needed to be considered as a factor which affected the change of velocity, as a result, he wrote the following equation:

$$\frac{\mathrm{d}v_{n+1}(\mathrm{t})}{\mathrm{dt}} = \kappa[V(\Delta x) - v_{n+1}(\mathrm{t})] + \lambda \Delta v \tag{1.7}$$

Based on this equation, he deduced a new momentum equation:

$$\frac{dv}{dt} = \frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} = \frac{v_e - v}{T} + \frac{\Delta}{\tau}\frac{\partial v}{\partial x}, \tag{1.8}$$

where $\frac{\Delta}{\tau}(= c)$ is the diffusion velocity of destabilization, T is the relaxation time of vehicle and v_e is the balance velocity of traffic flow. He examined his model and proved that this model fitted the real traffic flow and solved Daganzo's problems [7].

Many other improved models have been proposed to depict the real traffic flow. However, they generally assume that the relaxation time T and the diffusion velocity of destabilization c are constant or only change with the density of traffic flow ρ, and they do not taken the proportion of self-driving, cooperating cars into consideration. As a result, we introduce a new variable: the proportion of self-driving, cooperating cars to research the effects of self-driving, cooperating cars on traffic flow, on the basis of these successful models.

Due to the fact that practical traffic flow is discrete, cellular automata can be applied to simulate this complicated process effectively, which was initially proposed by Von Neuman [8] in 1950s. In 1980s, S. Wolfram [9] did further research on cellular automata and made it become a scientific methodology. He also proposed a simple cellular automata model: Wolfram184 in 1983. In 1986, Cremer and Ludwig [10] firstly applied the theory of cellular automata into traffic flow research.

In 1992, Nagel and Schreckenberg [11] put forward the successful and famous NaSch model on the basis of Wolfram 184, which is a one dimensional cellular automata. In this model, the speed of vehicle ranges from 0 to the maximum of the speed and cars will follow the following steps [12]:

(1)Speed-up: V_n = max(,V_n+1, V_{max}), drivers tend to drive at the fastest speed;

(2)Slow-down: V_n = min(,V_n,d_n), in order to avoid any collision, drivers will slow it down if needed;

(3)Randomly Slow-down: V_n = max(,V_n-1,0), in real life, drivers have to decelerate because of some random reasons;

(4)Movement: $x_{n+1} = x_n + V_n$, the next state of a car.

In 1992, Biham, et al [13] put forward a two dimension cellular automata ---BLM model, which was applied to analyze the complex city transportation. BML model uses a $\mathrm{N} \times \mathrm{N}$ matrix and suppose that all the cars are either heading for the north or the east and each cell has a traffic light. When it is odd number time, the south-north direction cars will move one step. If it is an even number, the west-east cars will move one step. Other rules are the same with one dimension cellular automaton [12].

NS model and BML model are two fundamental cellular automata in this field. Though they are very simple and cannot be applied to simulate real and complex systems, they are the basis of varieties of cellular automata and lay a solid foundation for modern cellular automata. For example, in 1993, M. Takayasu and H. Takayasu [14] proposed TT model, which introduced a slow-to-start possibility. In 1998, Nagel [15] introduced a TCA model with multiple lanes, adding the laws of change lanes. In 1999, Chowdhury, et al [16] proposed a new two dimensional TCA model---CS model, which included the effects of signal lights and had more practical meanings.

These improved models mainly concentrate on introducing new principles in order to make cellular automata more corresponding to real transportation system. However, they lack further research on the cooperation and interaction among vehicles, especially self-driving cars. As a result, we propose some new rules to abstract the cooperation between self-driving cars as well as the interaction between self-driving and non-self-driving vehicles, aimed at reveal the inner laws of modern traffic flow.

2. Problem Restatement

Traffic capacity is limited in many regions of the United States due to the fact that the number of lanes of roads is constant but the traffic flow becomes increasingly dense. For example, in the Greater Seattle area drivers experience long delays during peak traffic hours because the volume of traffic exceeds the designed capacity of the road networks. This is particularly pronounced on Interstates 5, 90, and 405, as well as State Route 520, the roads of particular interest for this problem.

Self-driving, cooperating cars have been proposed as a solution to increase capacity of highways without increasing number of lanes or roads. The behavior of these cars interacting with the existing traffic flow and each other may help solve this problem. However, it is not well understood because the mechanism of cooperation between self-driving cars as well as the interaction between self-driving and non-self-driving vehicles is still unclear. As a result, the Governor of the state of Washington has asked for analysis of the effects of allowing self-driving, cooperating cars on the roads listed above in Thurston, Pierce, King, and Snohomish counties.

Our work ought to concentrate on the effects of self-driving, cooperating cars on the current traffic flow and we need to establish mathematical models to provide solutions for the following several questions:

- Establish mathematical models which takes the number of lanes, peak and/or average traffic volume, and percentage of vehicles using self-driving, cooperating systems into consideration, and apply this model to analyze the effects of these variables on traffic flow.
- Depict how the effects change as the percentage of self-driving cars increases and find out whether there exist equilibrium point or tipping point.
- Reveal the mechanism of cooperation between self-driving cars as well as the interaction between self-driving and non-self-driving vehicles.
- Apply the actual data provided in attached spreadsheet to test our model and improve it.
- Offer some valuable suggestions on the policy concerning self-driving,

cooperating cars to the Governor of the state of Washington.

3. Problem Analysis

Though there are large quantities of factors leading to the congestion of traffic flow, we can classify them into avoidable factors and unavoidable factors. Unavoidable factors cannot be predicted or controlled, such as extreme weather, traffic accident (exclude accidents caused by drivers' illegal or irresponsible behaviors) and construction project of roads, which even self-driving, cooperating cars cannot avoid. By contrast, avoidable factors are mainly caused by human's controllable behavior or vehicles' system problems, which can be predicted or controlled. As a result, we need to analyze the relation among self-driving, cooperating cars and these avoidable factors. Significantly, we concentrate on human's controllable behavior and analyze how self-driving cars affect this set of factors so as to analyze the effects on traffic flow.

We assume that non-self-driving cars can only acquire the traffic information within drivers' vision, which we define as in-horizon information (IHI). And drivers need reaction time to understand them. Though one driver's reaction time is short, when they add up, they can lead to chain reaction in traffic flow, leading to phenomena such as phantom jam and pileup. In addition, drivers are easily affected by their metal states. When their metal states are unstable, they are more likely to cause traffic accidents, influencing traffic flow negatively. Moreover, drivers do not always observe traffic laws strictly and they are likely to make mistakes, which also affect traffic flow negatively. In contrast, self-driving cars can not only acquire IHI, but also acquire traffic information beyond drivers' vision, which we define as out-horizon information (OHI). According to OHI, self-driving cars can adjust their moving state globally. Beside this, self-driving cars do not need reaction time and they nearly do not make mistakes. In other words, they are skilled in processing emergency and they will not break laws rigorously.

Based on the above analysis, we assume that cooperation between self-driving cars can offer valuable OHI to optimize vehicles' states. And interaction between self-driving and non-self-driving vehicles can provide accurate IHI for self-driving cars, which can eliminate the negative effects of unstable human behaviors on traffic flow. We determine to introduce the proportion of self-driving, cooperating cars to make quantitative analysis on this problem and we determine to reveal the effects of self-driving, cooperating cars from both a macroscopic perspective based on hydrodynamics and a microscopic perspective based on cellular automata theory.

In the following parts of article, we will represent self-driving, cooperating cars with SDV and non-self-driving cars with NSDV.

4. Symbols and Assumptions

4.1 Symbols

Symbol	Meaning	Unit
ρ	The density of traffic flow in one lane	one car per meter
ρ_m	The maximal density of traffic flow	one car per meter
v	The velocity of traffic flow	m/s
v_m	The nominal speed limit of highway	m/s
v_e	The balance velocity of traffic flow	m/s
T	The relaxation time of vehicle	s
c	The diffusion velocity of destabilization	m/s
c_m	The minimal diffusion velocity of destabilization	m/s
t_0	The operation time	s
t_1	The reaction time of vehicle	s
α	The proportion of self-driving, cooperating cars	
g	The source item	one car per meter per second
γ	The influence coefficient of source item	
N	The number of lanes	
ρ_n^t	The difference element of ρ	one car per meter
v_n^t	The difference element of v	m/s
g_n^t	The difference element of g	one car per meter per second
IHI	In-horizon information	Note: In the following parts of paper, if we do not write a unit after a symbol, its unit refers to this table.
OHI	Out-horizon information	
SDV	Self-driving, cooperating cars	
NSDV	Non-self-driving cars	
AV	Average velocity of traffic flow	
RLV	Rate of low-speed vehicles	
FSB	Frequency of slamming on breaks	
FCL	Frequency of changing Lanes	

4.2 General Assumptions

• On average, 8% of the daily traffic volume occurs during peak travel hours.

• The nominal speed limit for all these roads is 60 miles per hour.

• Lane widths are the standard 12 feet.

• The parameters of highway are constant.

• The traffic flow is only effected by the parameters of roads and the characteristics of vehicles.

• We only consider single traffic flow instead of mixed traffic flow. As a result, we assume that all vehicles have the same qualities on the highway, which means that their kinetic parameters and their size are constant, and they can make same decisions under same conditions.

• We assume that NSDVs can only acquire IHI and there are no cooperation or interaction with other vehicles. Moreover, they cannot acquire the information of back vehicles.

• We assume that SDVs can acquire the location and velocity of any surrounding vehicles accurately. They can also acquire the location and velocity of vehicles beyond their vision by the interaction among SDVs.

• We assume that there are no signal lights on the highway and vehicles can only enter into or exit the highway through ramps.

5. Model Ⅰ: MDTF

5.1 Brief Introduction

Our first model is named as Modern Dynamic Model of Traffic Flow (MDTF). We improve former researchers' model by introducing a new variable: the proportion of SDVs and we analyze the effects of allowing SDVs in two conditions.

5.2 Dynamic Equations of This System

Inspired the continuity equation and momentum equation of hydrodynamics, we can get the dynamics equations of traffic flow. Considering that there are large quantities of ramps on the highway, the traffic volume is dynamic in real time. Therefore, we add a source item $\frac{g}{N}$ on the right side of continuity equation:

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho v)}{\partial x} = \frac{g}{N}, \tag{5.1}$$

where v is the velocity of traffic flow, ρ is the density of traffic flow in one lane, and the unit of g is one car times per meter times per time.

When deducing momentum equation, we refer to a new dynamic model of traffic model proposed by Rui Jiang, et al [7]:

$$\frac{dv}{dt} = \frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} = \frac{v_e - v}{T} + c\,\frac{\partial v}{\partial x} - v\frac{g}{\rho N}\gamma\,, \tag{5.2}$$

where T is the relaxation time of vehicle, c is the diffusion velocity of destabilization and v_e is the balance velocity of traffic flow.

Furthermore, we assume that ramps on the highway influence the traffic flow negatively. As a result, we introduce parameter γ to analyze the negative effect of this item and we assume that the intersection among SDVs will make it decrease. We apply a nonlinear function to depict the effect of self-driving, cooperating cars:

$$\gamma = 1 + \frac{1}{1 + e^{-0.5}} - \frac{1}{1 + e^{-(\alpha - 0.5)}}, \tag{5.3}$$

where α is the proportion of SDVs.

According to Castillo and Benitez's research [17], the balance velocity of traffic flow can be written in an equation form:

$$v_e = v_m \left(1 - exp\left(1 - exp\left(\frac{c_m}{v_m}\left(\frac{\rho_m}{\rho} - 1\right)\right)\right)\right), \tag{5.4}$$

where v_m is the nominal speed limit of highway, c_m is the minimal diffusion velocity of destabilization when facing traffic jam and ρ_m is the maximal density of traffic flow.

In order to take the effect of SDVs into consideration, we assume that the relaxation time of vehicle is related to the p proportion of SDVs. Wesheng An [18] proposed that relaxation time was composed of reaction time and operation time, and reaction time would decrease when the density of traffic flow increased. He also defined a nonlinear function to describe the relation between T and ρ. Inspired by this principle, we define the following function to depict the relation among T, ρ and α:

$$T = t_1(1 - \alpha)e^{\frac{\rho_m - \rho}{\rho_m}} + t_0, \tag{5.5}$$

where t_0 is the operation time and t_1 is the reaction time of vehicle.

Moreover, concerning that the diffusion velocity of destabilization will drop if the driver has low reaction sensitivity, we view that the proportion of SDVs will also influence the diffusion velocity of destabilization and we define the following function:

$$c = c_0 - c_1 e^{-\alpha}, \tag{5.6}$$

where c_0 and c_1 are constants with positive value.

In conclusion, we establish the following dynamic model of traffic flow to research the effect of SDVs on traffic flows.

$$\begin{cases} \dfrac{\partial \rho}{\partial t} + \dfrac{\partial(\rho v)}{\partial x} = \dfrac{g}{N} \\ \dfrac{\mathrm{d}v}{\mathrm{dt}} = \dfrac{\partial v}{\partial \mathrm{t}} + v\dfrac{\partial v}{\partial x} = \dfrac{v_e - v}{T} + \mathrm{c}\,\dfrac{\partial v}{\partial x} - v\dfrac{g}{\rho N}\gamma \\ \gamma = 1 + \dfrac{1}{1+e^{-0.5}} - \dfrac{1}{1+e^{-(\alpha-0.5)}} \\ v_e = v_m\left(1 - \exp\left(1 - \exp\left(\dfrac{c_m}{v_m}\left(\dfrac{\rho_m}{\rho} - 1\right)\right)\right)\right) \\ T = t_1(1-\alpha)e^{\frac{\rho_m - \rho}{\rho_m}} + t_0 \\ c = c_0 - c_1 e^{-\alpha} \end{cases} \tag{5.7}$$

In addition, to solve and research this dynamic system, we apply finite difference method to acquire its difference form:

$$\begin{cases} \rho_n^{t+1} = \rho_n^t + \dfrac{\Delta t}{\Delta x}\rho_n^t(v_n^t - v_{n+1}^t) + \dfrac{\Delta t}{\Delta x}v_n^t(\rho_{n-1}^t - \rho_n^t) + \dfrac{g_n^t}{N}\,. \\ v_n^{t+1} = \begin{cases} v_n^t + \dfrac{\Delta t}{\Delta x}(c_0 - v_n^t)(v_{n+1}^t - v_n^t) + \dfrac{\Delta t}{T}(v_e - v_n^t) - v_n^t\dfrac{g_n^t}{\rho_n^t N}\gamma\,, \ v_n^t < c \\ v_n^t + \dfrac{\Delta t}{\Delta x}(c_0 - v_n^t)(v_n^t - v_{n-1}^t) + \dfrac{\Delta t}{T}(v_e - v_n^t) - v_n^t\dfrac{g_n^t}{\rho_n^t N}\gamma\,, \ v_n^t \geq c \end{cases} \end{cases}$$

(5.8)

We employ MATLAB to solve these difference equations.

5.3 Situation Ⅰ of Model Ⅰ: Diffusion of Traffic Jam

(1)Highway conditions: Assume that there are no ramps on the highway, which means that the source item g is equal to zero. General assumptions are also effective in this situation. In this situation, we set a traffic jam in the middle of this highway and we simulate the diffusion of this traffic jam.

(2)Boundary conditions: The density of traffic flow ρ and the velocity of traffic flow v are constant on the boundary, which will not change with time (Riemann Boundary).

(3)Initial conditions: When $t = 0$, the density of traffic flow ρ satisfies Gauss distribution (because Gauss function is smooth and has an evident peak) and the velocity of traffic flow v is equal to balance velocity v_e:

$$\rho_{t=0} = \rho_1 + \rho_2 * e^{-(\frac{x-\mu}{\sigma})^2}, v_{t=0} = v_e \tag{5.9}$$

(4)Simulation: $\rho_1 = 0.08, \rho_2 = 0.06, \rho_m = 0.2, \mu = 250, \sigma = 20, v_m = 30, c_m = 8, t_0 = 1, t_1 = 4, c_0 = \frac{24}{3}, c_1 = \frac{8}{3}$, the length of highway is 50km, the total time is 3600s, the location step is 100m and the time step is 1 s.

5.4 Results and Analysis of Situation Ⅰ

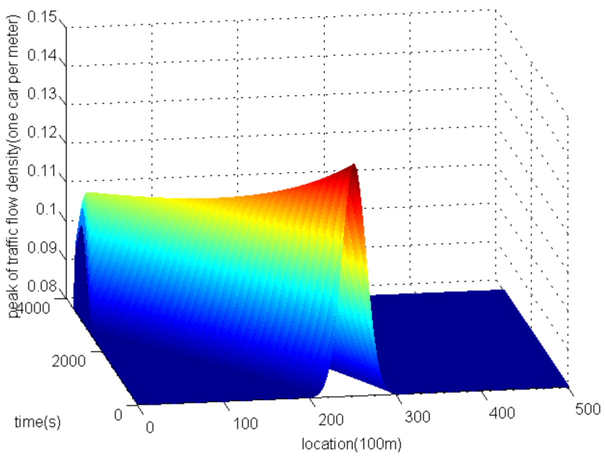


Figure 5.1 The diffusion of traffic jam in an hour (α = 0.5)

As shown in figure 5.1, the traffic jam will move backward with time increasing, and the peak of traffic flow density will drop slowly with time adding up, which corresponds to the real situations and shows the evolution of traffic jam in a macroscopic perspective.

Then we change the proportion of SDVs to research the evolution of traffic jam. As illustrated in figure 5.2, the peak of traffic flow drops more quickly if there are more SDVs, the tendency of which is significant.

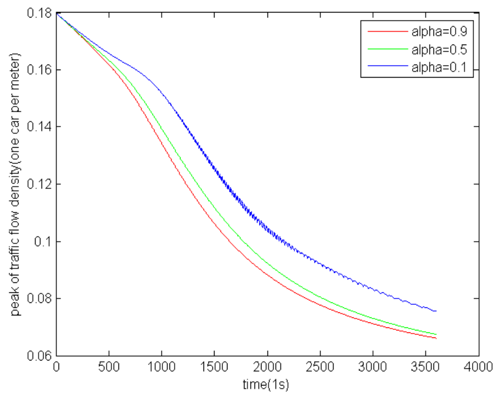
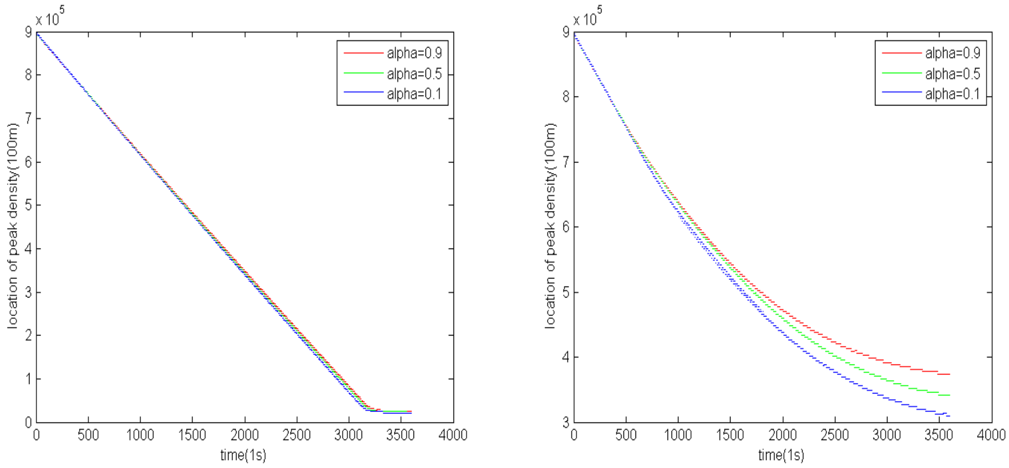


Figure 5.2 The change of the peak of traffic flow density with time in different α

Figure 5.3 (1) shows that the diffusion velocity of traffic jam will decrease with the increase of SDVs. Though this tendency is slight, it reflects that self-driving, cooperating cars can lighten the effects of traffic jam on the back road. And when traffic jam becomes more serious (as shown in Figure 5.3 (1), where $\rho_1 = 0.04, \rho_2 = 0.14$), this phenomenon will become increasingly significant.



(1) (2)

Figure 5.3 The change of the location of peak density with time in different α

5.5 Situation Ⅱ of Model Ⅰ: Take Ramps into Consideration

(1)Highway conditions: In this situation, general assumptions are also effective . We do not set a traffic jam, instead we assume that there is a ramp on the highway, which means that the traffic volume will increase with time. To smooth the source item g, we assume that g satisfies Gauss distribution because of its evident peak, which is similar to increase of traffic flow caused by ramp. The source item g has the following form:

$$g = g_0 * e^{-(\frac{x-\mu}{\sigma})^2} \tag{5.10}$$

(2)Boundary conditions: The density of traffic flow ρ and the velocity of traffic flow v are constant on the boundary, which will not change with time (Riemann Boundary).

(3)Initial conditions: When $t = 0$, the density of traffic flow ρ is constant and the velocity of traffic flow v is equal to balance velocity v_e:

$$\rho_{t=0} = \rho_0 , v_{t=0} = v_e . \tag{5.11}$$

(4)Simulation: $\rho_m = 0.2, \mu = 250, \sigma = 20 , v_m = 30, c_m = 8, t_0 = 0.5, t_1 =$

4.5, $c_0 = 14, c_1 = 6, N = 3,$ the length of highway is 50km, the total time is 2000s, the location step is 100m and the time step is 0.25 s.

5.6 Results and Analysis of Situation Ⅱ

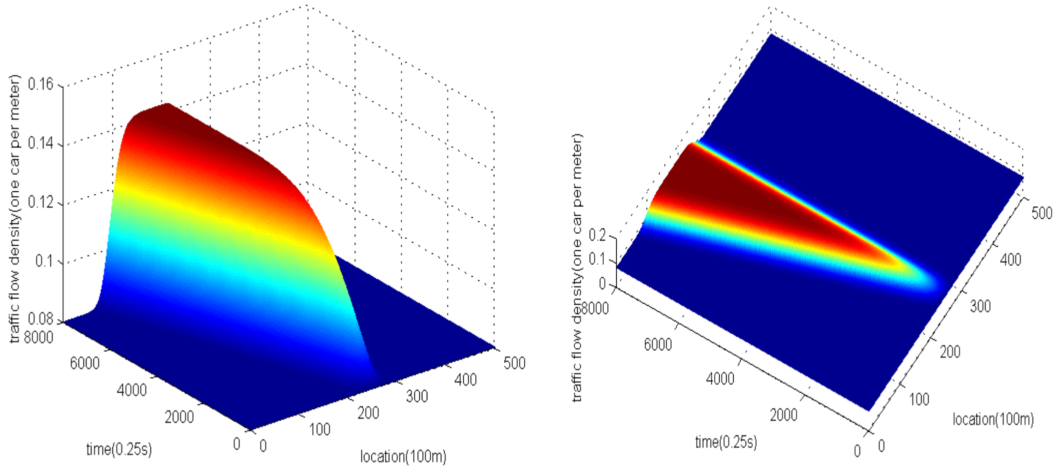


Figure 5.4 The effects of one ramp in 2000s ($\alpha = 0.5, \rho_0 = 0.08, g_0 = 0.002$)

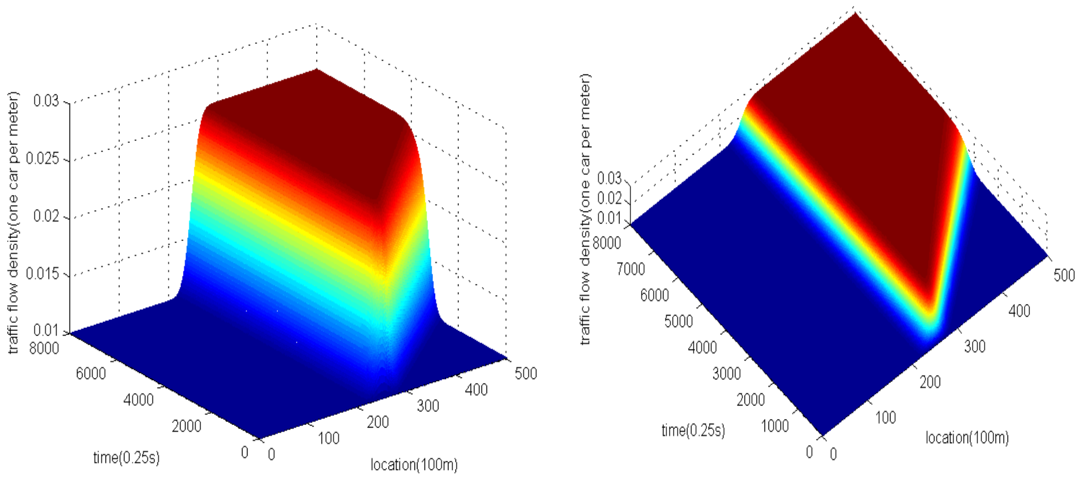


Figure 5.5 The effects of one ramp in 2000s ($\alpha = 0.5, \rho_0 = 0.01, g_0 = 0.002$)

As shown in figure 5.4 and figure 5.5, the density of traffic flow will increase when there is one ramp on the highway. Given the source item g_0, when the initial density of traffic flow ρ_0 is relative low, the density of traffic flow after the ramp will add up reasonably, which means that the ramp will not lead to traffic jam. However, when the initial density of traffic flow ρ_0 is relative high, the density of traffic flow before the ramp will add up significantly, which means that the ramp will lead to traffic jam.

Then we change the proportion of SDVs to research the evolution of traffic flow.

As illustrated in figure 5.6, the increase of SDVs can make the traffic flow smoother and lighten the peak traffic flow density. However, this tendency is not very evident compared to situation Ⅰ.

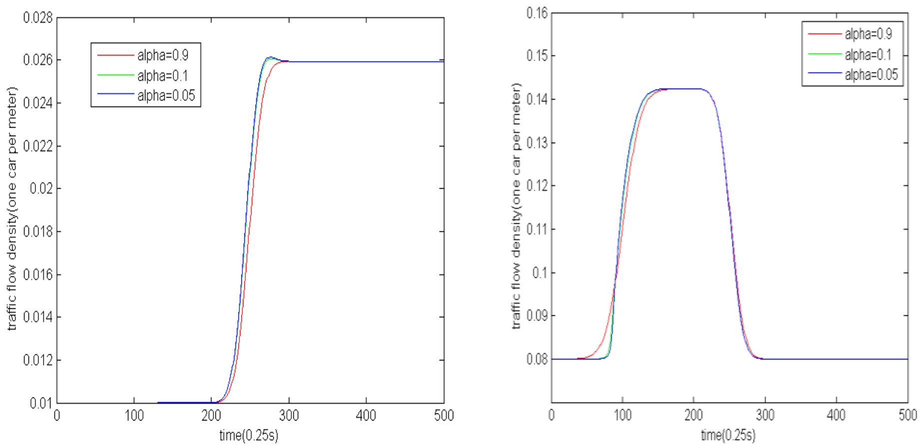


Figure 5.6 The change of traffic flow density with location at end in different α

5.7 Conclusions of Model Ⅰ

In situation Ⅰ, we find that SDVs are effective to release the pressure of traffic jam and lighten the chain reaction of traffic jam. In this situation, with the increasing number of self-driving, cooperating cars, the traffic flow become more stable. This conclusion corresponds to reality because SDVs take less time to adjust their moving state with more accuracy, thus the traffic jam caused by human's behavior can be lightened obviously.

In situation Ⅱ, we find that SDVs are effective to smooth the density distribution of traffic flow when there exists one ramp on the way by releasing the negative effect of source item g on the change of velocity of traffic flow. This conclusion corresponds to reality because SDVs have a comprehensive understanding of the whole road, as result, when facing ramps, they can optimize their state globally.

Therefore, we hold the view that SDVs are effective in releasing the traffic pressure on the highway mainly by reducing the reaction time of vehicles and it is reasonable to allow SDVs instead of limiting them.

6. Model Ⅱ: STCA

6.1 Brief Introduction of Model Ⅱ

Our second model is named as Smart Traffic Flow Model Based on Cellular Automata (STCA). We employ cellular automata to simulate the traffic flow mixed with SDVs. What's more, we take both cooperation between SDVs and the interaction between SDVs and NSDVs into consideration. Lastly, we add a ramp in our model, which will generate vehicles randomly.

6.2 Definitions of Index

- **Range of vision (RV):** Vehicles can only acquire IHI within the range of RV.
- **Safe distance (SD):** In normal conditions, distance between two adjacent vehicles in one lane is bigger that safe distance when moving. The safe distance can be defined as follows:

$$\mathrm{SD} = \begin{cases} v_1\Delta t + \dfrac{v_1^2 - v_2^2}{2a} + S_0 \,, if\ v_1\Delta t + \dfrac{v_1^2 - v_2^2}{2a} > 0 \\ S_0 \,,\ \ if\ v_1\Delta t + \dfrac{v_1^2 - v_2^2}{2a} \le 0 \end{cases} , \tag{6.1}$$

where v_1 is the velocity of current vehicle, v_2 is the velocity of front vehicle, Δt ie the reaction time of human (for self-driving cars, $\Delta t = 0$), a is the normal acceleration of vehicles on the road and S_0 is the minimal distance between two vehicles (cannot be avoided).

- **Synchronic distance (SYD):** When the distance between two adjacent vehicles reaches SYD (an extremely dangerous distance), they will have the same velocity so as to avoid collision. We define it to depict one kind of interaction between two SDVs based on the assumption that SDV has a precise controlling system.
- **Average velocity of traffic flow (AV):** It is the average velocity of vehicles on the whole road and it can describe the condition of the road in a macroscopic view.
- **Frequency of changing Lanes (FCL):** When a vehicle changes lanes successfully, the value of FCL will add one time. It can reflect the order of the whole road and we assume that higher FCL means the road is less ordered and less smooth.
- **Rate of low-speed vehicles (RLV):** When the velocity of a vehicle is smaller than 20miles/h, we can view it as low-speed vehicle. It can reflect the difference among vehicle on the road and describe the condition of the road in another view, compared with AV.
- **Frequency of slamming on breaks (FSB):** To simplify the model, we introduce FSB to depict frequency of emergencies on the road, which is another significant index when evaluating the condition of the road.

6.3 Two Fundamental Laws

- **Law of moving:**

(1) The vehicle makes trial acceleration and the new velocity v_1^{n+1} is equal to the minimum of $v_1^n + a_0$ and v_{max}.

(2) If both this vehicle and its front vehicle are SDVs and they are within SYD, v_1^{n+1} will be equal to v_2^n and move until their distance reaches S_0. Otherwise, it will calculate SD according to v_2^n. If the distance S is smaller than SD, it will try to decelerate at normal acceleration a_0 and move. And if it will collide with the front car, it will slam on the break, making FSB add one time.

(3) If the vehicle is NSDV, two random numbers are generated to simulate faults people may make: one is called randomization [11], which suggests people have possibility p_1 to decrease the speed randomly while moving; the other one is called slow-to-start model [19], which suggests people's possibility of delay in acceleration form standing states to moving states. According to these two random numbers, we will change vehicles' velocity.

- **Law of changing lanes**

(1) In order to move fast, the vehicle will change their lanes. When changing lanes, we first consider changing to the left lane. Then we consider changing to the right lane.

(2) Take changing to the left lane for example, where several conditions need to be met (in order) before changing lanes: ①The front vehicle is within RV;②The front vehicle on the left lane is further the front vehicle;③The front vehicle on the left lane is out of SD;④If the vehicle is SDV, the distance between it and the back vehicle on the left lane need to be smaller than SD.

(3) If all these conditions are satisfied, the vehicle will change their lanes.

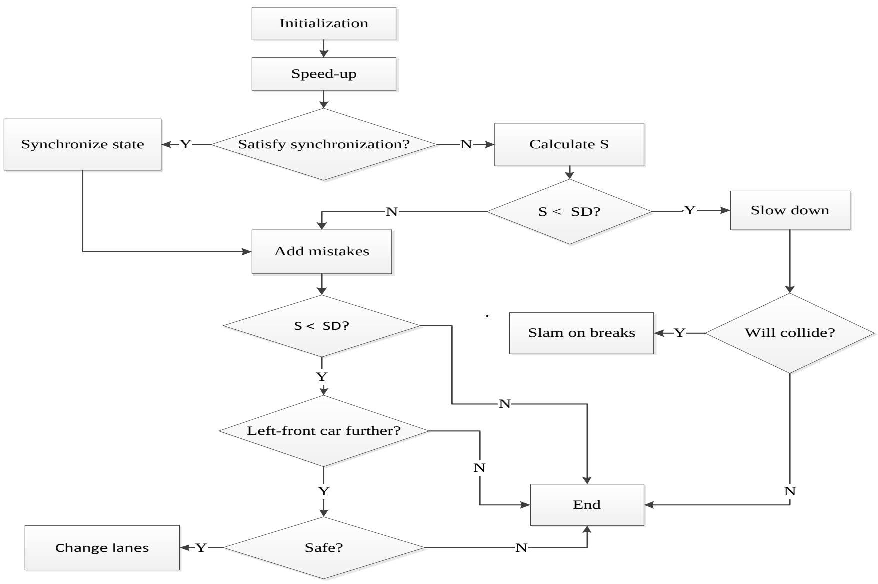


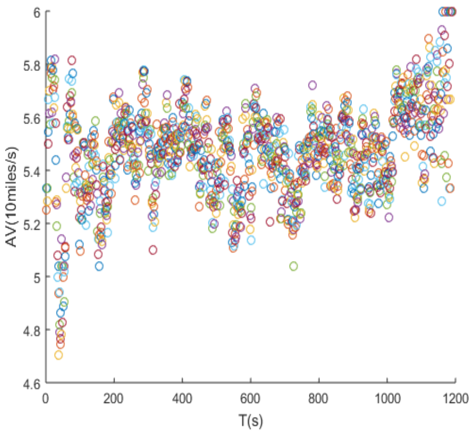
Figure 6.1 Flow chart of our STCA

6.4 Differences between SDV and NSDV in our laws

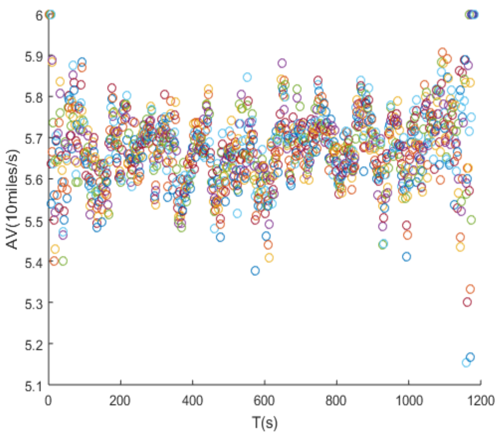
- **Smaller SD:** In our model, SDV has no reaction time, which means that they can adjust their moving state in real time. This capacity is critical to transportation system because the existence of human's reaction time accounts for large quantities of negative traffic phenomena, such as phantom jam and traffic accidents.
- **No mistake:** In our model, SDV will not make mistakes because we assume that the controlling system can always keep its working state, which means that two classical random mistakes happening in human will lose their effects for SDV.
- **Synchronous speed in SYD:** In our model, we proposed SYD to process emergencies on the highway. When the distance between two adjacent vehicles reaches a dangerous value suddenly, NSVD will slam on their breaks, which may lead to unpredictable chain reaction on the highway. However, the interaction between two adjacent SDVs can make their share moving information with each other. As a result, they can have the same velocity and realize perfect synchronization.
- **OHI:** NSDVs can only acquire the information of their front vehicles with RV, which means that they cannot determine whether their behaviors will affect vehicles behind them. By contrast, SDVs can acquire OHI by the interaction among SDVS, which means that they will consider the effects of their behaviors on surrounding vehicles instead of just the front vehicles.

6.5 Results of Simulation

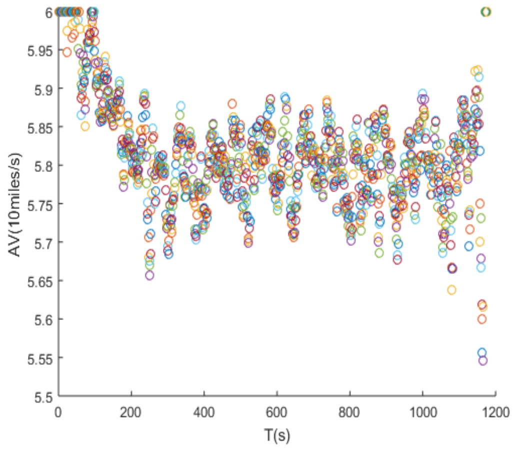
- Comparison of AV in different proportion of SDVs



$\alpha = 0.1$



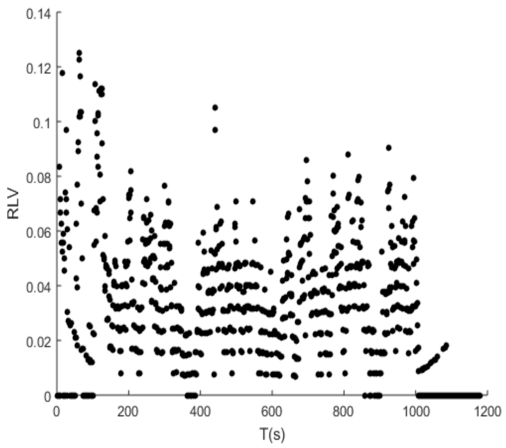
$\alpha = 0.5$



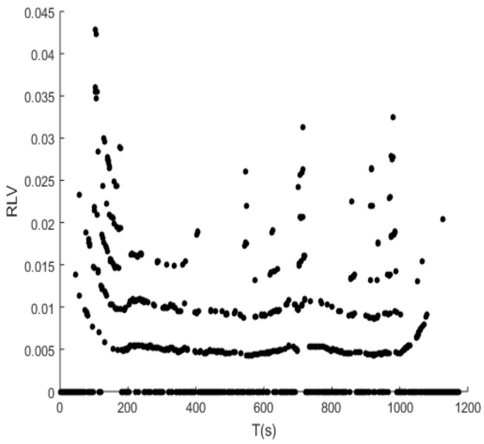
α = 0.9

Figure 6.2 AV in different $\alpha(N = 3, v_m = 60mile/h,$ *2000 vehicles and 1000s)*

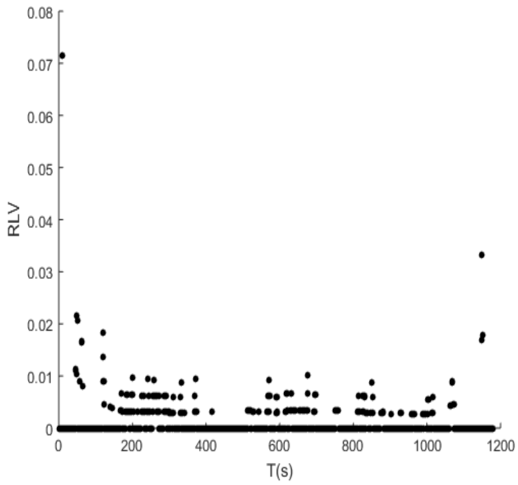
- Comparison of RLV in different proportion of SDVs



α = 0.1



α = 0.5



α = 0.9

Figure 6.3 RLV in different $\alpha(N = 3, v_m = 60mile/h,$ *2000 vehicles and 1000s)*

- Comparison of FSB in different proportion of SDVs

α	0.1	0.3	0.5	0.7	0.9
FSB	785	474	242	129	45

Table 6.1 FSB in different $\alpha(N = 3, v_m = 60 mile/s,$ *2000 vehicles and 1000s)*

- Comparison of FCL in different proportion of SDVs

α	0.1	0.3	0.5	0.7	0.9
FCL	1554	972	413	173	25

Table 6.2 FCL in different $\alpha(N = 3, v_m = 60 mile/s,$ *2000 vehicles and 1000s)*

6.6 Conclusions of Model Ⅱ

As shown in figure 6.2, when the proportion of SDVs increases, the average velocity of the vehicle will add up significantly, which means that the transportation system become more efficient. As illustrated in figure 6.3, rate of low-speed vehicles will drop obviously with increasing number of SDVs, which means that the traffic flow become more smooth and comfortable. Table 6.1 and table 6.2 show that both the value of FSB and the value of FCL will drop if there are more SDVs on the road, which means that there are less traffic accidents or emergencies on the highway.

As a result, we can conclude that allowing SDVs on the highway is an effective solution to traffic jams. We also prove that both the cooperation between SDVs and the interaction between SDVs and NSDVs have positive effects on traffic flow and our laws reveal their mechanism to some extent.

7. Sensitivity Analysis

7.1 Finding tipping point and equilibrium point

Based on our STCA model, we do further research on the relationship between the proportion of SDVs and the quality of traffic flow, by changing the proportion of SDVs in a small step, in order to curve the functional relation.

However, we do not find evident tipping point and equilibrium point. Instead, we find that there exists a linear relation between AV and α and the absolute value of slope in RLV (α), FCL (α) and FSB (α) will drop with SDVs add up. This is reasonable because we do not consider the drawbacks of SDVs and we view them as a perfect system with no mistakes.

We employ curve fitting toolbox of MATLAB to depict the best curve. The results are as follows:

Evaluation Index	Fitting curve	R(correlation coefficient)
AV	$y = 0.009571x + 5.03$	0.9723
RLV	$y = 15.76e^{-(\frac{x+43.06}{59.81})^2}$	0.9933
FCL	$\mathrm{y} = 1950\mathrm{e}^{-(\frac{x+25.38}{54.11})^2}$	0.9945
FBS	$\mathrm{y} = 917.6\mathrm{e}^{-(\frac{x+16.11}{54.91})^2}$	0.9883

Table 7.1 Curve fitting results *($N = 3, v_m = 60mile/h$, 2000 vehicles and 1000s)*

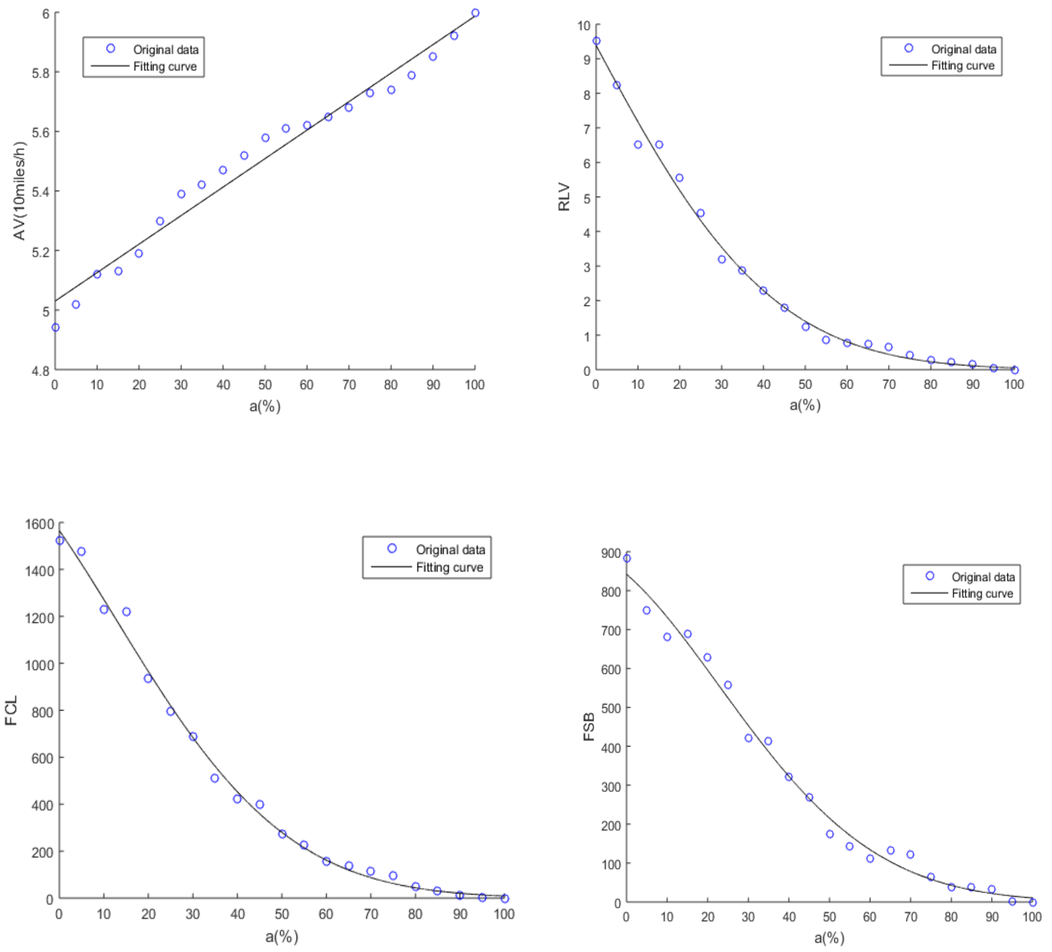


Figure 7.1 Curve fitting results *($N = 3, v_m = 60mile/h$, 2000 vehicles and 1000s)*

7.2 Applying Our Model to Route 5, 90 405 and 520

To testify whether our model correspond to real statistics, we find some valuable from the website of Washington State Department of transportation. According to ***Washington State Speed Report*** in four quarter of calendar year 2015, we choose one site which is near the urban area of Thurston, Pierce, King, and Snohomish counties in route 5, 90 405 and 520 relatively. The basic information of the sites we choose is list in table 7.2 and their geographic locations are marked on figure 7.2.

Site ID	Site Name	State Route	Mile Post	Lane by Dir. Inc/Dec	Speed Limit	Country Name
S837	Fife	5	136.80	4/4	60	Pierce
R117	Seattle/Rainier	90	3.00	4/4	60	King
S824	Bothell	405	28.99	3/4	60	Snohomish
D10	Toll Plaza	520	4.00	2/2	50	King

Table 7.2 Site Indexes (the unit of velocity is mile/h)

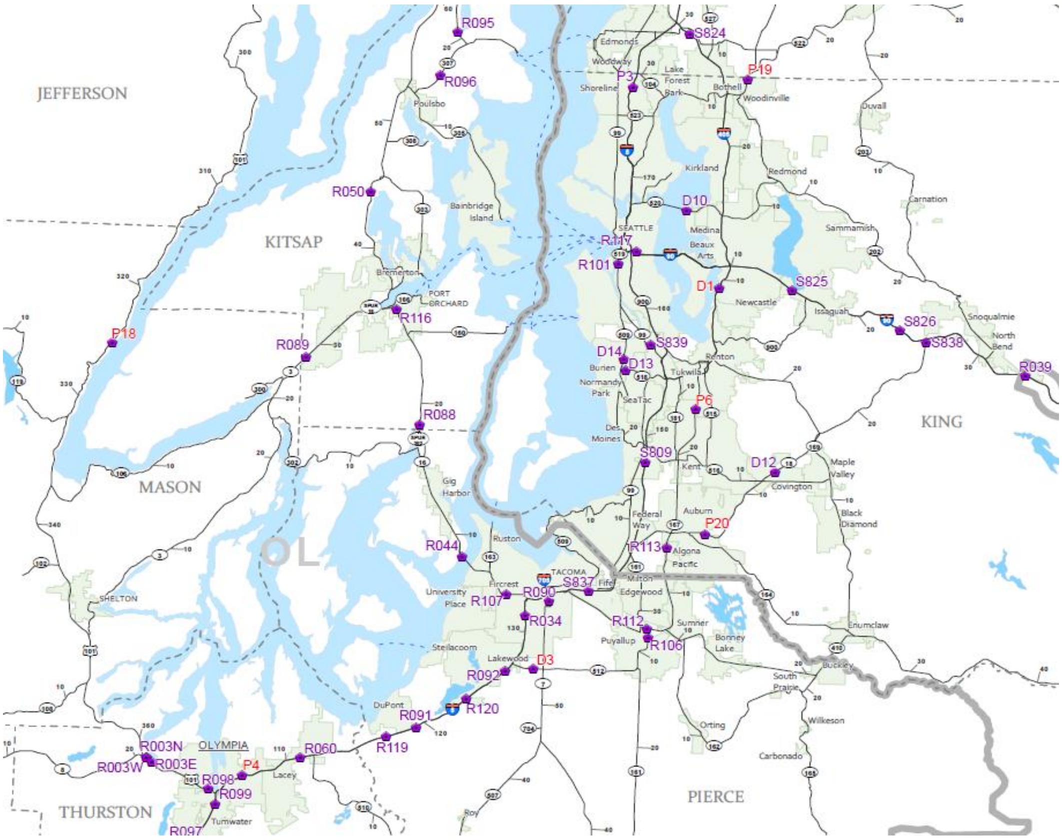


Figure 7.2 Site Locations

We utilize the data of roads (see in table 7.3) near these four sites in the Excel spreadsheet as the input parameters of our model and get the results. We first compare AV with real data. Then we adjust our parameters to make them correspond to the practical situation. Finally, we calculate the AV when SDVs adding up. We find that our models can easily satisfy the real data and it is effective to allow SDVs in Washington State, which is evident in these four sites. The results are shown in table 7.4.

State Route	startMilepost	endMilepost	Average daily traffic counts Year_2015	Average daily flow-in traffic	Lane by Dir. Inc/Dec
5	136.51	137.15	193000	-12000	4/4
90	2.79	3.94	121000	98000	4/4
405	27.4	29.88	119000	28000	3/3
520	1.63	4.4	68000	7000	2/2

Table 7.3 Data of roads near these four sites (the unit of velocity is mile/h)

Site ID	Official Data Inc/Dec	Model Result (α = 0)	Model Result (α = 0.1)	Model Result (α = 0.5)	Model Result (α = 0.9)
S837	62/53	56.9	57.1	57.6	59.4
R117	56/52	54.2	54.5	56.8	59.1
S824	54.5/58	56.4	56.8	57.7	59.3
D10	52.5/47	49.9	50.1	50.3	50.9

Table 7.4 Comparison between AV from real data and AV simulated by STCA
(the unit of velocity is mile/h)

8. Strengths and Weaknesses

8.1 Strengths

- **Two perspectives**

We establish two model to explain the effects of SDVs on traffic flow from two distinguishing perspective. Our first continuous model gives an explanation from a macroscopic perspective and our second discrete model gives an explanation from a microscopic perspective.

- **Improved models**

Though our models are based on former researchers' work, we apply them to describe a new phenomenon: SDVs. We also improve their work and put forward some new rules to expand their research areas. For example, in model I, we propose that the proportion of SDV will change the relaxation time. In model II, we propose synchronic distance to depict the interaction among SDVs.

- **Various situations**

In model I, we simulate two classic traffic phenomena to research the effects of SDVs: diffusion of traffic jam and effects of ramps. In model II, we abstract several simple and new laws from a variety of conditions to demonstrate the mechanism of the cooperation between SDVs and the interaction between SDVs and NSDVs.

- **Applying models to real data**

We not only apply our STCA model to simulate and explain realistic problems theoretically, but also apply it to real data and offer some valuable information and suggestions.

8.2 Weaknesses

- **Model I**

When solving difference equations in model I, we do not guarantee their stability and do research on the diffusion of their error, which may make this dynamic system easily affected by unpredictable turbulence. Moreover, we do not gather practical data to curve parameters in our equations; instead, we just simulate two theoretical situations. In further research, we ought to give a more rigorous deduction of this dynamic system and we need to improve it with practical data.

- **Model II**

SDVs in our STCA model are idealized and they also have the possibility of making mistake and being unstable. As a result, we need to take SDVs' negative effects into consideration in further research.

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A Letter to the Governor of the state of Washington

Mr. Governor:

We are very glad to receive your letter asking for our advice on self-driving, cooperating cars on the Washington State!

Recently, we establish two models to analyze the traffic flow mixed with self-driving, cars (**SDV**) and non-self-driving cars (**NSDV**). The first one is Modern Dynamic Model of Traffic Flow (**MDTF**) based on the conservation laws in hydrodynamics, which analyzes the effects of SDVs on traffic flow in a macroscopic perspective. And the second one is Smart Traffic Flow Model Based on Cellular Automata (**STCA**), which simulates the effects of SDVs on traffic flow in a microscopic perspective.

According to our research results of model one, we conclude that SDVs are effective to release the pressure of traffic jam , smooth the density distribution of traffic flow and lighten the chain reaction of traffic jam. Based on the simulation results of model two, we conclude that the average velocity of vehicles, the rate of low-speed vehicles the frequency of changing lanes and the frequency of slamming on breaks all will drop with increasing number of SDVs on the road, which means that there are less traffic accidents or emergencies on the highway.

As a result, we strongly suggest that Washington State should allow SDVs instead of restricting them, due to the fact that transportation system is of great significance to the development of human society and community economy in contemporary world and a highly efficient transportation system can accelerate the manufacturing development of a country remarkably. What's more, with the rapid development of artificial intelligence, especially the theory of deep learning, SDV will become increasing mature and powerful. As a result, they will definitely make far less mistake that human and enhance the efficiency of transportation system greatly. In addition, allowing SDV on the road will also promote the development of SDV manufacturing so as to inject new vitality to local economy, isn't it a perfect thing?

We hope that our suggestions are of some value to you!

Yours sincerely

XXXXXX

XXXXXX

Appendix

A. Code of MDTF:

```
function [ dd ] = TrafficFlow( max_v, max_d, up_d, down_d, alpha, t1, t2, c1, ch)
%parameters setting
delta_t = 1;    %time step
delta_l = 100;    %distance step
max_t = 3600;    %total time
max_l = 500;    %total distance
d = zeros(max_t,max_l);    %matrix of flow density
v = zeros(max_t,max_l);    %matrix of flow velocity
c = c1 * (2 / 3 + 1 / 3 * (2 - exp(- alpha)))    %propagation velocity ofdestabilization
d0 = zeros(1,max_l);
v0 = zeros(1,max_l);
%initialize boundary conditions
for i = 1:max_l
        d0(1,i) = down_d + (up_d - down_d) *    exp(- ((i - max_l /2)/20)^2);
        v0(1,i) = balv(max_v, c1, max_d, d0(1,1)) * d0(1,1) / d0(1,i);
end
for i = 1:max_t
        v(i,max_l) = v0(1,max_l);
end
d0
%solve difference equations
for j = 1:max_t
        d(j,1) = d0(1,1);
        v(j,1) = v0(1,1);
        for i = 2:max_l-1
                if j == 1
                        d(j,i) = d0(1,i) + delta_t/delta_l * d0(1,i) * (v0(1,i) - v0(1,i+1)) +
delta_t/delta_l * v0(1,i) * (d0(1,i-1) - d0(1,i));
                else
                        d(j,i) = d(j-1,i) + delta_t/delta_l * d(j-1,i) * (v(j-1,i) - v(j-1,i+1)) +
delta_t/delta_l * v(j-1,i) * (d(j-1,i-1) - d(j-1,i));
                end
                if j == 1
                        ve = balv(max_v, c1, max_d, d0(1,i));
                        t = t1 + t2 * (1 - alpha) * exp(1 - d0(1,i)/max_d);
                        if v0(1,i) < c
                                v(j,i) = v0(1,i) + delta_t/delta_l * (c - v0(1,i)) * (v0(1,i+1) -
v0(1,i)) + delta_t / t * (ve - v0(1,i));
                        else
                                v(j,i) = v0(1,i) + delta_t/delta_l * (c - v0(1,i)) * (v0(1,i) - v0(1,i-1))
```

```
+ delta_t / t * (ve - v0(1,i));
                end
            else
                ve = balv(max_v, c1, max_d, d(j-1,i));
                t = t1 + t2 * (1 - alpha) * exp(1 - d(j-1,i)/max_d);
                if v(j-1,i) < c
                    v(j,i) = v(j-1,i) + delta_t/delta_l * (c - v(j-1,i)) * (v(j-1,i+1) - 
v(j-1,i)) + delta_t / t * (ve - v(j-1,i));
                else
                    v(j,i) = v(j-1,i) + delta_t/delta_l * (c - v(j-1,i)) * (v(j-1,i) - 
v(j-1,i-1)) + delta_t / t * (ve - v(j-1,i));
                end
            end
        end
        d(j,max_l) = d(j,max_l-1);
        v(j,max_l) = v(j,max_l-1);
end
for j = 1:max_t
        tt(1,j) = j * delta_t;
        for i = 1:max_l
            z(j,i) = d(j,i);
            x(1,i) = i * delta_l;
        end
end
%Visualization
[X,T] = meshgrid(x,tt);
mesh(z);
d;
for i = 1:max_t
        plot(i, max(d(i, 1:max_l)),ch);
        hold on;
end
for i = 1:max_t
        dd(1,i) = max(d(i, 1:max_l));
end
end
```

B. Code of STCA:

```
Maxlen=300;
Maxlane=3;
road=zeros(Maxlen,Maxlane);
temproad=zeros(Maxlen,Maxlane);
Totalnum=2000;
Totaltime=1000;
```

```
Times=zeros(Totalnum);
Sdpropotion=1;
Maxv=6;
Jamv=2;
a=2;
t=1;
Pemergency=0;
Pfault1=0.1;
Pfault2=1;
Pleak=0;
Leakpos=Maxlen/2;
carnum=0;
v=zeros(Totalnum);
type=zeros(Totalnum);
static=zeros(Totalnum);
Seerange=3*Maxv;
Synrange=Maxv/6;
totalchange=0;
totalem=0;
Schange=0;
time=0;
jamnum=0;
figure;
for i=1:Totalnum
    Times(i)=floor(rand()*Totaltime)+1;
end
Times=sort(Times);
vx=0;nx=0;
sumv=0;sumn=0;
while(time<3*Totaltime)
    time=time+1;
    fprintf('%d\n',time);
    %car in
    if(carnum<Totalnum)
        temp=0;

while(temp<=Maxlane*2&&carnum<=Totalnum-1&&Times(carnum+1)<=time)
            templane=floor(1+rand()*Maxlane);
            temp=temp+1;
            if(road(1,templane)==0)
                carnum=carnum+1;
                road(1,templane)=carnum;
                temp3=rand();
                if(temp3<1)
```

```
                    v(carnum)=Maxv;
                else
                    v(carnum)=Maxv/2;
                end
                temp=rand();
                if(temp<Sdpropotion)
                    type(carnum)=1;
                end
            end
        end
    end
%     fprintf('%d%d\n',temptime,time);
%     for j=1:Maxlane
%         for i=1:100
%             fprintf('%d',road(i,j));
%         end
%         fprintf('\n');
%     end
    %move
    temproad=zeros(Maxlen,Maxlane);
    for i=1:Maxlen-1
        for j=1:Maxlane
            if(road(i,j)==0)
                continue;
            end
            sd=0;
            tempv=v(road(i,j));n=road(i,j);
            %speed up
            v(n)=min(v(n)+1,Maxv);
            %speed down
            pre=i+1;
            while(pre<=Maxlen&&road(pre,j)==0)
                pre=pre+1;
            end

if(pre<=Maxlen&&pre-i<=Synrange&&type(n)==1&&type(road(pre,j))==1)
                v(n)=v(pre);sd=min((pre-i-1)/1.5,3);
            else
                if(pre<=Maxlen)
                    v1=v(n);v2=v(road(pre,j));
                    if(type(n)==1)
                        s=max(0,floor((v1^2-v2^2)/(2*a)));
                    else
                        s=max(0,floor(v1*t+(v1^2-v2^2)/(2*a)));
```

```
                    end
                    if(s>pre-i)
                        v(n)=max(v(n)-a,0);
                        if(v(n)>=v(road(pre,j))+pre-i)
                            v(n)=max(0,pre-i-1);
                            totalem=totalem+1;
                        end
                    end
                end
            end
            %fault1
            temp=rand();
            if(type(n)==0&&temp<Pfault1&&tempv>0)
                v(n)=max(floor(v(n)-2),0);
            end
            %fault2
            if(v(n)==0)
                static(n)=1;
            end
            temp=rand();

if(type(n)==0&&temp<Pfault2&&tempv==0&&v(n)>0&&static(n)==1)
                static(n)=0;
                v(n)=0;
            end
            %change
            temp1=rand();

if(i+v(n)+sd<=Maxlen&&(temp1>Pleak||i+v(n)+sd<Leakpos||i>Leakpos))
                temproad(i+v(n)+sd,j)=n;
            end
            if(i+v(n)>=982&&i<982)
                vx=vx+v(n);
                nx=nx+1;
            end
        end
    end
    road=temproad;
    %change lane
    for j=1:Maxlane
        for i=1:Maxlen
            if(road(i,j)>0&&type(road(i,j))==0)
                test=1;n=road(i,j);
                if(j>1)
```

```
pre1=i+1;
while(pre1<=Maxlen&&road(pre1,j)==0)
    pre1=pre1+1;
end
pre2=i+1;
while(pre2<=Maxlen&&road(pre2,j-1)==0)
    pre2=pre2+1;
end
next1=i-1;
while(next1>=1&&road(next1,j-1)==0)
    next1=next1-1;
end
if(pre1-i<=Seerange&&pre2>pre1)
    v1=v(n);
    if(pre2<=Maxlen)
        v2=v(road(pre2,j-1));
    else
        v2=Maxv;
    end
    if(type(n)==1)
        s=max(0,floor((v1^2-v2^2)/(2*a)));
    else
        s=max(0,floor(v1*t+(v1^2-v2^2)/(2*a)));
    end
    if(type(n)==1)
        if(next1==0)
            v1=0;
        else
            v1=v(road(next1,j+1));
        end;
        v2=v(n);
        if(next1>=1&&type(road(next1,j+1))==1)
            ss=max(0,floor((v1^2-v2^2)/(2*a)));
        else
            ss=max(0,floor(v1*t+(v1^2-v2^2)/(2*a)));
        end
    end

if((s<pre2-i&&type(n)==0)||(type(n)==1&&pre2-i>s&&i-next1>ss))
        road(i,j-1)=n;
        road(i,j)=0;
        test=0;
    end
end
```

```
        end
        if(test&&j<Maxlane)
            pre1=i+1;
            while(pre1<=Maxlen&&(road(pre1,j))==0)
                pre1=pre1+1;
            end
            pre2=i+1;
            while(pre2<=Maxlen&&road(pre2,j+1)==0)
                pre2=pre2+1;
            end
            next1=i-1;
            while(next1>=1&&road(next1,j+1)==0)
                next1=next1-1;
            end
            if(pre1-i<=Seerange&&pre2>pre1)
                v1=v(n);
                if(pre2<=Maxlen)
                    v2=v(road(pre2,j+1));
                else
                    v2=Maxv;
                end
                if(type(n)==1)
                    s=max(0,floor((v1^2-v2^2)/(2*a)));
                else
                    s=max(0,floor(v1*t+(v1^2-v2^2)/(2*a)));
                end
                if(type(n)==1)
                    if(next1==0)
                        v1=0;
                    else
                        v1=v(road(next1,j+1));
                    end;
                    v2=v(n);
                    if(next1>=1&&type(road(next1,j+1))==1)
                        ss=max(0,floor((v1^2-v2^2)/(2*a)));
                    else
                        ss=max(0,floor(v1*t+(v1^2-v2^2)/(2*a)));
                    end
                end

if((s<pre2-i&&type(n)==0)||(type(n)==1&&pre2-i>s&&i-next1>ss))
                    road(i,j+1)=n;
                    road(i,j)=0;
                    test=0;
```

```
                end
            end
        end
        temp2=rand();
        if(temp2<0.1)
            totalchange=totalchange+1;
            if(test==0)
                Schange=Schange+1;
            end
        end
      end
    end
  end
%   for j=1:Maxlane
%       for i=1:Maxlen
%           fprintf('%d',road(i,j));
%       end
%       fprintf('\n');
%   end
%   fprintf('\n');

%   %pos-time figure
%   for j=1:Maxlane
%       %plot([1,Maxlen],[2*time+(j-1)/Maxlane,2*time+(j-1)/Maxlane]);
%       hold on;
%       for i=1:Maxlen
%           if(road(i,j)>0)
%               if(type(road(i,j))==0)
%                   scatter(i,2*time+(j-1)/Maxlane,30,'k','filled');
%                   hold on;
%               else
%                   scatter(i,2*time+(j-1)/Maxlane,30,'b','filled');
%                   hold on;
%               end
%           end
%       end
%   end
  %v-time figure
  totalv=0;num=0;
  for j=1:Maxlane
    for i=1:Maxlen
      if(road(i,j)>0)
        totalv=totalv+v(road(i,j));
        num=num+1;
```

```
                if(v(road(i,j))<=Jamv)
                    jamnum=jamnum+1;
                end
            end
        end
    end
    if(num>0)
        sumn=sumn+num;
        sumv=sumv+totalv;
    end
    %fprintf('%d %d %d\n',time,totalv,num);
%     if(totalv/num>5.2)
%     scatter(time,totalv/num);
%     end
    %scatter(time,jamnum/num,20,'k','filled');
    %hold on;
end
xlabel('T(s)');ylabel('AV(10miles/h)');
fprintf('%d\n',sumv/sumn);
fprintf('%d\n',jamnum/sumn);
fprintf('%d\n',Schange);
fprintf('%d\n',totalem);
```